Try these:

① $\lim_{p \to \infty} \dfrac{\sin^5(2p)}{e^{p^5}-1}$

② $\lim_{x \to 2} \dfrac{2^x-4}{\sin(x-2)}$

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① $\lim_{p \to \infty} \dfrac{\sin^5(2p)}{e^{p^5}-1} = 0$

between -1 & 1

$e^{p^5} \to \infty$

Note $-1 \le \sin^5(2p) \le 1$

② $\lim_{x \to 2} \dfrac{2^x-4}{\sin(x-2)} = \lim_{x \to 2} \dfrac{2^x-4}{\frac{\sin(x-2)}{(x-2)} \cdot (x-2)}$

$\frac{\sin(x-2)}{(x-2)} \to 1$

$\frac{0}{0}$ form

$= \lim_{x \to 2} \dfrac{2^x-4}{(x-2)} = \lim_{x \to 2} \dfrac{2^{(x-2)+2}-4}{(x-2)}$

$= \lim_{x \to 2} \dfrac{2^{(x-2)} \cdot 2^2 - 4}{(x-2)}$

$= \lim_{x \to 2} \dfrac{2^{(x-2)} \cdot 4 - 4}{(x-2)}$ ($1 \cdot 4$)

$= \lim_{x \to 2} \dfrac{(2^{x-2}-1)4}{(x-2)}$

$A^B A^C = A^{B+C}$

$2^3 2^4$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 2^7$

$$= \lim_{x\to 2}\left(\frac{(e^{\ln 2})^{(x-2)} - 1}{x-2}\right)\cdot 4$$

$e^{\ln 2} = 2$

$$= \lim_{x\to 2} \frac{\left(e^{(\ln 2)(x-2)} - 1\right)}{(\ln 2)(x-2)} \cdot 4 \cdot (\ln 2) = 4\ln 2$$

$(\ln 2)(x-2) \to 0$ ↘ 1

Using Special limit

$$\lim_{y\to 0} \frac{e^y - 1}{y} = 1$$

$y = (\ln 2)(x-2)$

$$A\cdot B + C\cdot B = (A+C)B$$

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(3) $$\lim_{g\to\infty}\left(\frac{g+3}{g}\right)^g = \lim_{g\to\infty}\left(1+\frac{3}{g}\right)^g$$

$\frac{3}{g} \to 0$

form 1^∞

$$\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t = e$$

$$= \lim_{g\to\infty}\left(1+\frac{1}{g/3}\right)^g$$

$$= \lim_{\theta \to \infty} \left(1 + \frac{1}{\theta/3}\right)^{\frac{\theta}{3} \cdot 3}$$

$$= \boxed{e^3}$$